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ON THE VELOCITY OF BUCKLE PROPAGATION IN A BEAM ON A NONLINEAR ELASTIC FOUNDATION

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Abstract—The paper revisits a simple beam model used by Chater *et al.* (1983, *Proc. IUTAM Symp. Collapse*. Cambridge University Press) to examine the dynamics of propagating buckles on it. It was found that, if a buckle is initiated at a constant pressure higher than the propagation pressure of the model (P_P), the buckle accelerates and gradually reaches a constant velocity which depends upon the pressure, while if it is initiated at P_P, the buckle propagates at a velocity which depends upon the initial imperfection. The causes for the difference are also investigated.

1. INTRODUCTION

Certain structures have a tendency to propagate a buckle once one is initiated. Perhaps the most important example is the *buckle propagation* along a sea pipeline. When a pipe undergoes an extra external pressure, the weakest section of the pipe may experience collapse first to initiate a local buckle, then driven by the pressure, the buckle can propagate along the pipe flattening it (Palmer and Martin, 1975). The lowest pressure which can sustain the propagation in a quasi-static, steady-state is known as the *propagation pressure* (P_P). It is especially significant since at any pressure below P_P, the buckle remains local, while at any pressure above P_P, the buckle once initiated will run dynamically over the whole length. Therefore predicting the propagation pressure has been a main subject for discussion in the past two decades (Kyriakides and Babcock, 1981; Kyriakides and Arikan, 1983; Chater and Hutchinson, 1984; Kyriakides *et al.*, 1984; Wierzbicki and Bhat, 1986; Jensen, 1988; Dyau and Kyriakides, 1993). It becomes known that, fundamental to developing a longitudinal propagation, is the N-like postbuckling path of the structures in cross-sections. A complete, comprehensive review of the problem has been given by Kyriakides (1993).

Considerable understanding has been reached regarding the mechanism of quasi-static propagation. However, there were few works reported about the dynamics. Kyriakides and Babcock (1979) had performed a series of dynamic experiments in a constant pressure environment of $P > P_P$. They observed that after initiation the buckle accelerates and quickly reaches a constant velocity that is a function of the pressure. However the function fails to go through the origin as shown in Fig. 1 [from Kyriakides and Babcock (1979)]. The reason for this is not well understood up to date. Recently, a transient finite element simulation has been presented (Song and Tassoulas, 1992), but unfortunately, the analysis was limited to the steady-state value of the velocity. On the other hand, Chater *et al.* (1983) had proposed a simple beam model to elucidate some of the general features of buckle propagation. The inertia effects were considered by using a dynamic, steady-state assumption. In the present paper, we revisit the beam model but extend our attention from the dynamic, steady-state to the whole, transient process of a buckling, including the localization.

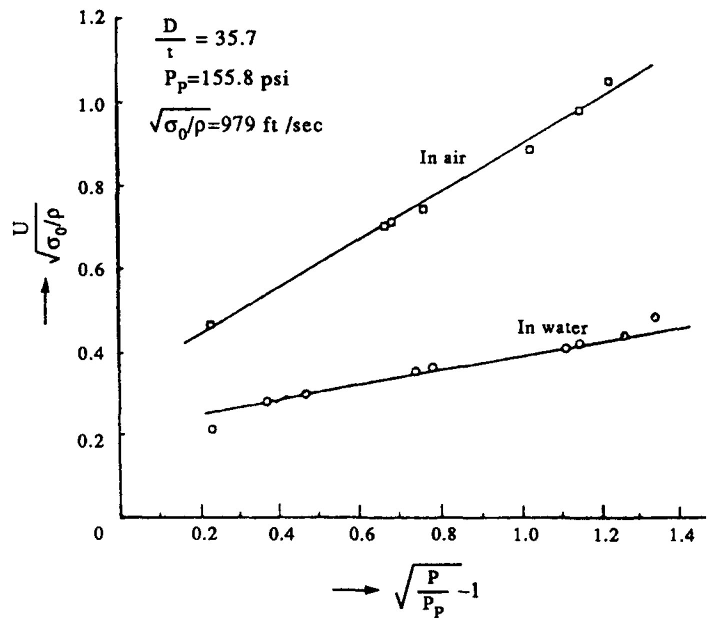

Fig. 1. Propagation velocity vs pressure parameter [from Kyriakides and Babcock (1979)].

2. MODEL ANALYSIS

The model used by Chater *et al.* (1983) is an infinite linear beam resting on a nonlinear elastic foundation according to

$$k(w) = k_0\left[1-4.5\left(\frac{w}{H}\right)+5.25\left(\frac{w}{H}\right)^2\right], \tag{1}$$

where w is the lateral deflection. The beam is subjected to a uniform lateral load p as depicted in Fig. 2. The restoring force per unit length of the foundation, $f(w) = k(w)w$, is assumed to have a general "N" shape shown in Fig. 3. It is this property that renders the model of some practical significance, especially for studying the longitudinal propagation characteristics.

The dynamic equation governing the system is

$$m\frac{\partial^2 w}{\partial t^2}+EI\frac{\partial^4 w}{\partial x^4}+k(w)w = p(t), \tag{2}$$

where EI is the flexural rigidity of the beam and m the mass of linear beam per unit length.

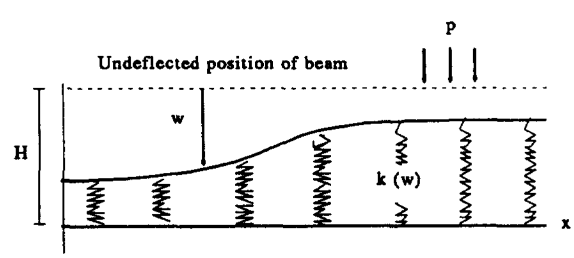

Fig. 2. The beam–foundation model used by Chater *et al.* (1983).

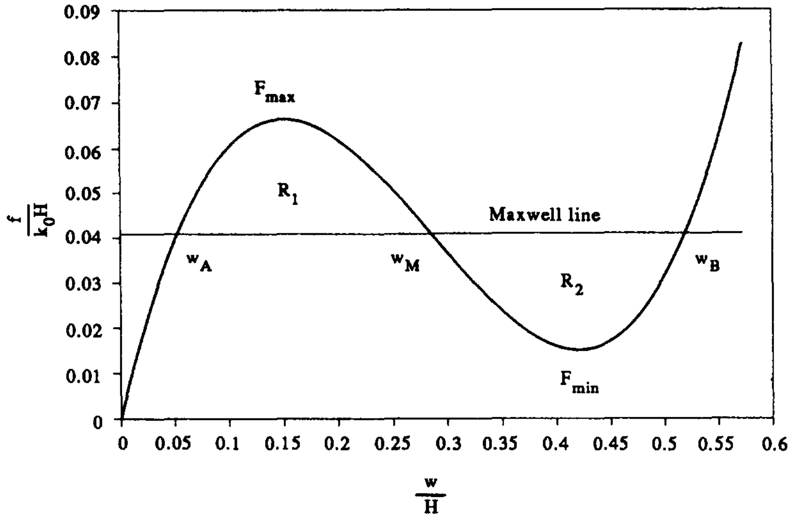

Fig. 3. Restoring force of the foundation vs deflection.

The boundary conditions are

$$x \to \pm\infty, \quad \frac{\partial w}{\partial x} = \frac{\partial^2 w}{\partial x^2} = 0 \tag{3}$$

and the initial conditions are

$$w(x,0) = W^* \tag{4a}$$

$$\frac{\partial w}{\partial t}(x,0) = 0, \tag{4b}$$

where W^* is the initial deflection of the beam. In accordance with the constant pressure conditions used in the dynamic experiments (Kyriakides and Babcock, 1979), we let

$$p(t) \equiv P^* \quad (-\infty < t < +\infty) \tag{5}$$

but assume the foundation is perfect until $t = 0$ and has the uniform deflection W^* under the action of P^*. Once $t > 0$, a weak spot (imperfection) at the vicinity of the origin on the foundation is manufactured to initiate a buckle;

$$k(w,x) = k_0\left[1-4.5\left(\frac{w}{H}\right)+5.25\left(\frac{w}{H}\right)^2\right][1-\eta\exp(-\lambda\xi^2)], \tag{6}$$

where

$$\xi = x\Big/\left(\frac{EI}{k_0}\right)^{1/4}.$$

The parameters η and λ are introduced to describe the weakness of the spot. With $\eta > 0$, the foundation is weakest near the origin and it develops its full strength for $\lambda\xi^2 \gg 1$. In the present paper two kinds of weakness are considered: (I) $\eta = 0.8$ and $\lambda = \frac{1}{6}$; (II) $\eta = 0.7$ and $\lambda = \frac{1}{12}$. Both are serious enough to initiate a buckle even at a pressure much lower than P_P.

The infinite beam is approximated by a finite one with a span of $2L$, $L \approx 180(EI/k_0)^{1/4}$, and subdivided into 180 elements, so that eqns (2)–(5) can be solved by a finite element method (details listed in the Appendix).

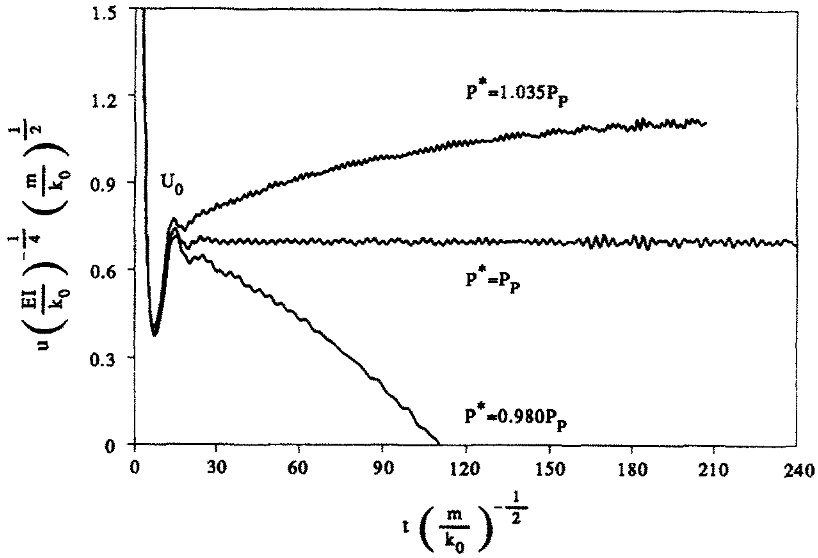

Fig. 4. Propagation velocity vs time (weakness I).

It is seen from Fig. 3 that the displacement $w/H = 0.15$, corresponds to the peak of the restoring force of the foundation. This indicates that once the deflection of the beam is somewhere beyond $0.15H$ then the point is going into buckling. Therefore the horizontal distance from this point to the origin of the foundation is defined as L_B in the present paper to locate the transient position where a buckle front propagates.

For each time increment $\Delta t = t^{n+1} - t^n$, two successive fronts could be obtained from computations: L_B^{n+1} and L_B^n. The mean velocity of propagation in Δt could be regarded as the transient velocity at $t^{n+1/2}$,

$$u^{n+1/2} = \frac{L_B^{n+1} - L_B^n}{t^{n+1} - t^n} \tag{7}$$

provided that the time interval Δt is sufficiently small.

3. VELOCITY OF PROPAGATION

The characteristics of the velocity of propagation on the model are illustrated by numerical examples in which the external pressure P^* is intentionally specified at three different constant levels but closely around the propagation pressure: (1) $P^* = 1.035P_P > P_P$; (2) $P^* = P_P$; (3) $P^* = 0.980P_P < P_P$. Although there is only a slight difference for the given pressures for the same initial weakness (weakness I), it causes a substantial difference in the velocity responses. Details will be described in the following two sections.

3.1. *Observations*

It is seen from the computational results shown in Fig. 4 that the time history of the velocity is generally divided into two periods. In the first period, no remarkable difference is observed among the curves and actually buckles do not propagate but have to complete initial localization. Although the velocities have lost their general meanings in this period, they reach an almost equal value U_0, finally. It will be known that U_0 is an initial velocity of propagation. Once time comes into the second period, however, the curves separate from each other.

(1) $P^* > P_P$. The buckle initiated in this case accelerates from U_0, and gradually tends towards a constant velocity U, $U > U_0$. The small waves along the curves in Fig. 4 are due to the computational errors, they could be eliminated by a finer discretization of the beam.

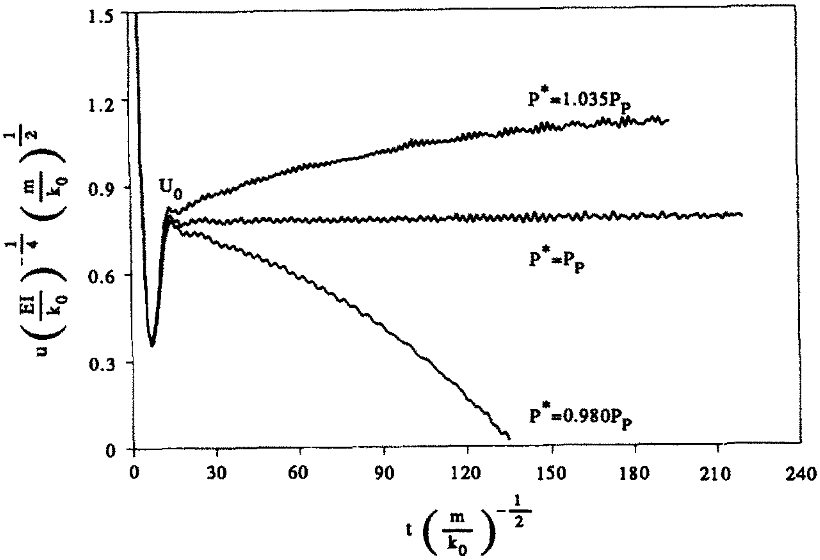

Fig. 5. Propagation velocity vs time (weakness II).

(2) $P^* = P_P$. The buckle propagates keeping the initial velocity U_0.

(3) $P^* < P_P$. The buckle decelerates from U_0 and, as a result, it cannot propagate over the full length of the beam. What was observed above is examined by another numerical example (weakness II, see Fig. 5).

Comparing Fig. 4 with Fig. 5 leads to:

(4) If $P^* > P_P$, then no matter how different the initial imperfection may be, an equal external pressure brings about the same steady velocity U (see Fig. 6), which indicates that the velocity U is only a function of the pressure.

(5) If $P^* = P_P$, then the velocity depends upon the imperfection; for instance, different weaknesses may induce different velocities (see Fig. 7). These conclusions are supported by other examples which are not shown in the paper.

It becomes clear that for the beam model considered, if a buckle is initiated at a pressure higher than P_P, it finally propagates at a constant velocity which depends upon the pressure. However, if it is initiated just at P_P, it propagates at a constant velocity which depends upon the initial imperfection.

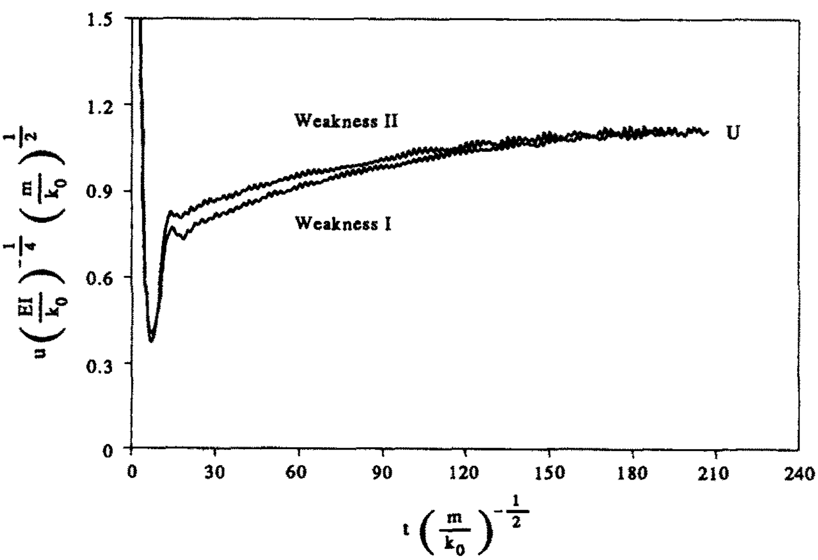

Fig. 6. U only depends upon the pressure ($P^* = 1.035P_P$).

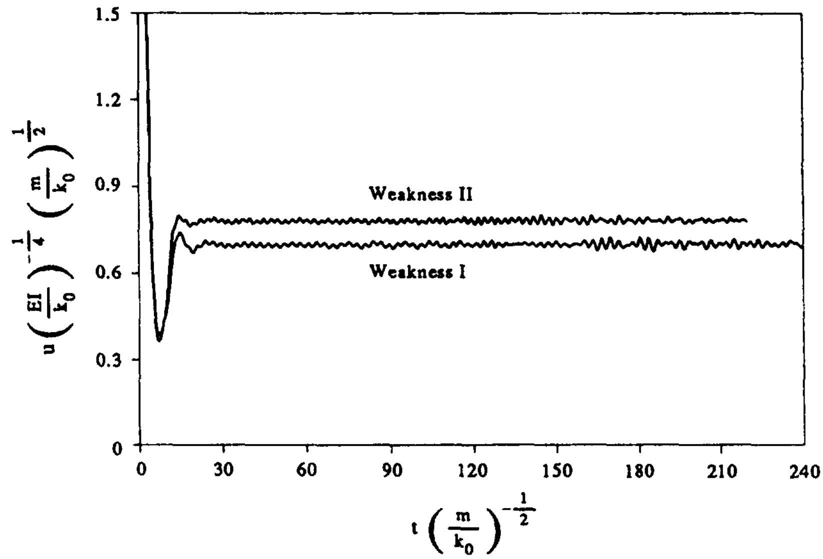

Fig. 7. The propagation velocity depends upon the imperfection ($P^* = P_P$).

3.2. *Explanations*

The cause that makes the velocity response vary in a very large variety at the propagation pressure will be discussed in the context of conservation of energy.

(1) $P^* > P_P$. The work done by the external pressure in this case is always consumed in two parts. The first part is turned into the elastic potential energy of the foundation as the beam collapses. The energy absorbed per unit length of the foundation E_P, is determined by the initial deflection W_A and the final deflection W_B. Therefore there is no essential difference between E_P at a dynamic state and that at a quasi-static state if the pressure only varies in a minor range around P_P (see Fig. 3). The second part $[\approx (P^* - P_P)(W_B - W_A)]$ is consumed for vibrating the beam at the final equilibrium position W_B. Since the buckled part of the beam (may be imagined as "mass blocks") and the elastic foundation below ("springs") construct a series of "oscillators" excited in sequence, the deflection of the buckled beam is no longer a constant, i.e. $\partial w/\partial x \neq 0$. This issue was ignored in the dynamic, steady-state analysis given by Chater *et al.* (1983). As the buckle propagates, more and more parts of the beam are accelerated. Once the rate of the second part of external work becomes equal to the rate of the kinetic energy which goes into the buckled beam, the propagation reaches a steady-state.

(2) $P^* = P_P$. The increment of the external work in this case is just right for increasing the potential energy of the foundation when a buckle propagates. No extra work could be obtained from the pressure to excite those "oscillators". Therefore the deflection of the beam falls at a constant lever behind the transition zone, i.e. $\partial w/\partial x = 0$. Only in this case the dynamic, steady-state assumption used by Chater *et al.* (1983) is valid. The total kinetic energy of the beam does not change either, it maintains its initial value as the buckle propagates. The initial kinetic energy is limited by the localization; its maximum, or the upper bound for U_0, can be estimated by using a weaker spot.

(3) $P^* < P_P$. The input work in this case is not enough for increasing the potential energy of the foundation. Thus, the initial kinetic energy is gradually used up to make up for the potential energy required to keep the buckle propagating, which results in the deceleration of the buckle. In other words, the buckle remains local.

4. DISCUSSIONS

What was observed resembles the dynamic behavior of the propagating buckles in submarine pipelines described at the beginning of the paper. However, the model examined in the present paper is a non-dissipative one, and the results obtained should be restricted to the model itself.

In submarine pipelines there are no "oscillators" to store the kinetic energy when a buckle propagates. Instead, the kinetic energy of the pipe is dissipated as the opposite walls impact each other. Once the rate of the kinetic energy dissipated reaches the rate of work supplied by the pressure, the buckle propagation comes to a steady state [see Kyriakides and Babcock (1979)]. Therefore the dynamic buckle propagation in elastic–plastic pipes is of a dissipative problem. A full understanding of this problem will be the next challenge.

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APPENDIX

Equation (2) is discretized by the finite element method as

$$\mathbf{M}\ddot{\mathbf{W}}+\mathbf{K}_S\mathbf{W} = p(t)\mathbf{P}, \tag{A1}$$

where $\mathbf{W}$ and $\ddot{\mathbf{W}}$ are nodal displacement and acceleration vectors, respectively. Both of them are functions of time. $\mathbf{M}$ denotes the mass matrix of linear beam and $\mathbf{K}_S$ the secant stiffness matrix for the beam–foundation system defined by $\mathbf{K}_S = \mathbf{K}+\mathbf{K}_F$, in which $\mathbf{K}$ indicates the stiffness matrix for the linear beam, and $\mathbf{K}_F$ the secant stiffness matrix for the nonlinear elastic foundation. For each beam element, $[K_F]_{4\times4}$ can be written as follows:

$$\begin{aligned}[K_F]_{ij} &= \int_0^l k(w,\bar{x})N_i(\bar{x})N_j(\bar{x})\,\mathrm{d}\bar{x} \\ &= C_0\beta^0_{ij}+C_1\beta^1_{ijk}W_k+C_2\beta^2_{ijmn}W_mW_n\end{aligned} \tag{A2}$$

with

$$\beta^0_{ij} = \int_0^l \alpha(\bar{x})N_iN_j\,\mathrm{d}\bar{x}$$

$$\beta^1_{ijk} = \int_0^l \alpha(\bar{x})N_iN_jN_k\,\mathrm{d}\bar{x}$$

$$\beta^2_{ijmn} = \int_0^l \alpha(\bar{x})N_iN_jN_mN_n\,\mathrm{d}\bar{x}$$

$$\alpha(\bar{x}) = 1-\eta\exp\left[-\lambda\sqrt{\frac{k_0}{EI}}(x_E+\bar{x})^2\right]$$

$$C_0 = k_0;\quad C_1 = -\frac{4.5}{H}k_0;\quad C_2 = \frac{5.25}{H^2}k_0, \tag{A3}$$

in which l is the length of the beam element, $N_i(\bar{x})$ is Hermite's shape function and x_E is the distance from the origin to the left node of each beam element. The Einstein summation convention is adopted for $i, j, k, l, m, n = 1, 2 \ldots 4$.

To integrate eqn (A1) with time, the Newmark constant acceleration method is employed as

$$w^{n+1} = w^n + \tfrac{1}{2}\Delta t(v^{n+1} + v^n)$$
$$v^{n+1} = v^n + \tfrac{1}{2}\Delta t(a^{n+1} + a^n), \tag{A4}$$

where v and a are velocity and acceleration of the beam, respectively. Combining the above two equations leads to

$$w^{n+1} = \tilde{w}^n + \tfrac{1}{4}(\Delta t)^2 a^{n+1}$$
$$\tilde{w}^n = w^n + \Delta t v^n + \tfrac{1}{4}(\Delta t)^2 a^n. \tag{A5}$$

Such a relationship brings eqn (A1) into a new form

$$(\mathbf{M}^{\mathrm{DYN}} + \mathbf{K}_S)\mathbf{W}^{n+1} = p^{n+1}\mathbf{P} + \mathbf{M}^{\mathrm{DYN}}\tilde{\mathbf{W}}^n, \tag{A6}$$

where

$$\mathbf{M}^{\mathrm{DYN}} = \frac{4}{(\Delta t)^2}\mathbf{M}.$$

For each time increment, the two terms on the right hand side of eqn (A6) are known, so the unknown variable $\mathbf{W}^{n+1}$ on the left hand side can be determined by the Newton–Raphson Method.